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ANALYSIS OF TWO SPECTRAL LINES WITH GAUSSIAN
PROFILE BY USING PHOTON STATISTICS.
COMPARISON WITH OTHER PROFILE SHAPES.

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ABSTRACT

In this paper we study the behavior of the errors involved in the determination of the parameters characterizing two spectral lines with Gaussian profile by means of the measurement of the intensity correlation function $g^{(2)}(T)$ and by a multichannel system. The results obtained with both methods are compared. We also compare this results with those obtained when two lines with Lorentzian profile are analyzed.

I.- INTRODUCTION

A procedure which is frequently used to improve the signal to noise when a light beam is being analyzed consists of repeating the experiment periodically and using a multichannel signal analyzer. When the intensity of the signal is very weak a photon-counting multichannel analyzer can be used. But there are other techniques which can also be used to process the signal. Photon statistics, in particular, is a powerful tool when light beams with fluctuating intensity are being studied¹⁻². This technique can also improve the signal to noise ratio when a deterministic signal is obtained³. In previous papers we used this technique to study the characteristics of light beams which are periodically repeated and applied it to some spectroscopic experiments⁴⁻⁵⁻⁶⁻⁷

In a work which will be published in Applied Spectroscopy in the September - October 1986 issue, we analyzed two spectral lines with Lorentzian profile by measuring the normalized intensity correlation function, $g^{(2)}(T)$, and by using a photon-counting multichannel analyzer. The behaviour of the errors involved in determining the width, sepa-

ration and relative heights of the two Lorentzian lines was studied.

In this work we have studied the behavior of the errors involved in determining the parameters (width, separation, and relative height) which characterize spectral lines with Gaussian profile by measuring $g^{(2)}(T)$ and by using a multichannel system. The results obtained with these two methods are compared. A Voigt profile (the convolution product of a Lorentzian and Gaussian profile) appears in many spectroscopic experiments; however, it is very difficult to characterize spectral lines with this profile directly because of the complexity involved in handling them numerically. We can now characterize each of the factors contributing to the Voigt profile by using the results reported in this paper and those obtained for Lorentzian profiles.

II.- METHOD

Let us consider a light beam whose spectrum has two lines to be analyzed. We shall assume that the two lines have Gaussian profiles and the same line-width. If the spectrum is periodically scanned the variation

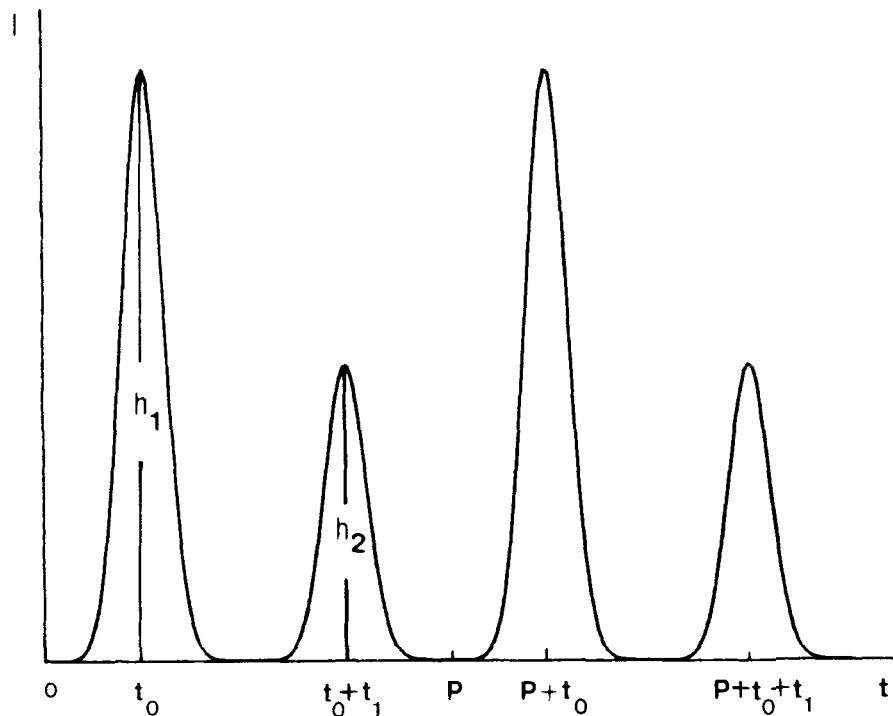


Figure 1.- Intensity I as a function of time t when scanning a two-line spectrum repeatedly with a period P .

of wavelength with time having a sawtooth profile with period P we will obtain a light beam with a periodic intensity $I(t)$ (Fig.1).

If a digital correlator is used with a counting time T , N samples of the number $n(t_i, T)$ of photopulses in a time interval $(t_i, t_i + T)$ are made in a nonsynchro-

nized way and the normalized intensity correlation function

$$g^{(2)}(jT) = \frac{\frac{1}{N} \sum_{i=1}^N n(t_i, T) n(t_i + jT, T)}{\left\{ \frac{1}{N} \sum_{i=1}^N n(t_i, T) \right\}^2} \quad (1)$$

is obtained for L channels. This function can be obtained theoretically from the expression⁸

$$g^{(2)}(jT) = \frac{\frac{1}{P} \int_0^P W(t, T) W(t + jT, T) dt}{\left\{ \frac{1}{P} \int_0^P W(t, T) dt \right\}^2} \quad (2)$$

where

$$W(t, T) = \int_t^{t+T} I(t') dt' \quad (3)$$

The unknown parameters can be determined by fitting the measured values of $g^{(2)}(T)$ to the theoretical ones.

If a photon-counting multichannel analyzer with a counting time T is used and $\theta_i (i=1, 2, 3, \dots, L)$; $0 \leq \theta_i \leq P$ is the instant when the ith channel starts the measurement of the values of $n(t_i, T)$ is synchronized

with the scanning process for M periods. Then, all the values that correspond to the same channel are averaged. By fitting these results to the calculated values of $W(\theta_i, T)$ the unknown parameters can be obtained.

In a work which will be published in Applied Spectroscopy in the September - October 1986 issue, we performed an experiment in which we proved the validity of the computer simulation method. This method consists of simulating the signal that is obtained from a digital correlator or from a photon-counting multichannel analyzer system, when analyzing a light beam with an intensity like that shown in Fig. 1. This was done by simulating values of $n(t, T)$ from the values of $I(t)$. To obtain the values of $n(t, T)$ the time interval T was divided into several subintervals in order to ensure that $I(t)$ was constant in each subinterval. Then a Poisson⁸ generator could be used to simulate the number of photoelectrons which correspond to each subinterval. The value of $n(t, T)$ could be obtained by adding the results for the corresponding subintervals. From the simulated values of $n(t, T)$ we obtained values of $g^{(2)}(T)$ and values for the signal that would have been obtained by a multichannel analyzer.

III.- RESULTS AND CONCLUSIONS

We studied periodic intensity profiles like those shown in Fig. 1 for different values of the half-width (Δt), the relative height ($h = h_2/h_1$) and the separation (t_1) of the two lines (we have assumed that $t_0 = (P - t_1)/2$.

The mean intensity was 100 photopulses per period. We used a counting time $T = 0.02P$ for $\Delta t/P > 0.1$ and $T = 0.01P$ for $\Delta t/P = 0.1$. The simulated values were calculated, taking the experiment time equal to $200P$. From each series of measurements the values of $\Delta t/P$, h and t_1/P were obtained. By repeating this process ten times the values of the relative errors involved in the determination of $\Delta t/P$ (e_{Δ}), h (e_h) and t_1/P (e_{t_1}) were evaluated. The results are shown in Tables I-III.

The behavior of the errors is roughly the same for the two techniques, keeping in mind that the errors involved in measurements made by a correlator system are slightly greater than those corresponding to a multichannel analyzer. The change in e_{Δ} when t_1 , h and Δt vary are small. The values of e_{t_1} decrease as t_1 or h increases ; they decrease when Δt decreases. The values of e_h decrease as t_1 increases (except for large

Table I.- Experimental values for e_h , e_A and e_{t1} obtained by using a digital correlator or a digital multichannel analyzer when $\Delta t/P \approx .2$, $h \approx .5$ and t_1/P is varied. The mean values obtained for $\Delta t/P$, h and t_1/P are also shown.

$\langle h \rangle$	e_h (%)	$\langle \Delta t/P \rangle$	e_A (%)	$\langle t_1/P \rangle$	e_{t1} (%)
.491	3.89	.199	2.06	.105	4.60
.502	1.86	.200	0.591	.300	0.501
CORRELATOR					
.501	1.87	.200	0.450	.495	0.840
.499	1.56	.200	0.390	.699	0.560
.494	1.71	.196	0.883	.892	0.487
MULTICHANNEL ANALIZER					
.502	4.55	.200	1.19	.102	4.20
.496	1.50	.199	.860	.300	0.566
.499	1.16	.199	.675	.499	0.227
.502	1.46	.199	.721	.699	0.149
.502	1.36	.201	.870	.900	0.190

Table II. - Experimental values for e_h , e_Δ and e_{t_1} obtained by using a digital correlator or a digital multichannel system analyzer when $\Delta t/P \approx .2$, $t_1/P \approx .3$ and h is varied. The mean values obtained for $\Delta t/P$, h and t_1/P are also shown.

$\langle h \rangle$	e_h (%)	$\langle \Delta t/P \rangle$	e_Δ (%)	$\langle t_1/P \rangle$	e_{t_1} (%)
.0985	3.54	.201	.886	.299	0.950
.302	1.35	.200	.811	.299	0.822
CORRELATOR					
.502	1.86	.200	0.591	.299	0.501
.704	1.92	.200	0.555	.300	0.313
.915	5.28	.200	0.508	.300	0.282
MULTICHANNEL ANALIZER					
.100	3.05	.199	1.02	.299	1.11
.299	1.80	.199	0.723	.300	0.600
.496	1.50	.200	0.860	.300	0.566
.697	1.52	.200	0.824	.300	0.308
.897	1.52	.199	0.848	.300	0.390

Table III.- Experimental values for e_h , e_Δ and e_h obtained by using a digital correlator or a digital multichannel system analyzer when $t_1/P \approx 3$, $h \approx .5$ and $\Delta t/P$ is varied. The mean values obtained for $\Delta t/P$, h and t_1/P are also shown.

	$\langle h \rangle$	e_h (%)	$\langle \Delta t/P \rangle$	e_Δ (%)	$\langle t_1/P \rangle$	e_{t_1} (%)
	.501	0.873	.100	0.340	.300	0.159
	.501	1.21	.150	0.657	.301	1.237
CORRELATOR	.502	1.86	.200	0.591	.299	0.501
	.497	1.78	.254	0.817	.300	0.650
	.509	5.09	.303	1.14	.301	1.02
	.496	1.06	.0995	0.729	.299	0.221
MULTICHANNEL ANALYZER	.497	1.46	.159	0.956	.300	0.325
	.496	1.50	.200	0.860	.300	0.566
	.499	1.59	.249	0.983	.300	0.777
	.486	1.42	.299	1.06	.300	0.706

values of t_1); they decreases as h increases. When a correlator is used e_h increases as Δt increases; for a multichannel system e_h changes little when Δt is varied.

If we compare the values of e_{Δ} , e_{t_1} and e_h obtained from a multichannel system with those obtained from correlator, we may define

$$f_{\alpha} = \frac{\text{error in } \alpha \text{ from } g^{(2)}(\tau)}{\text{error in } \alpha \text{ from multichannel analyzer}} \quad (4)$$

The results are compiled in Table IV.

It can be observed that Δt and t_1 (except for large values of t_1) can be obtained with a smaller errors when obtained from a correlator than when obtained from a multichannel analyzer. The opposite occurs for h .

Consequently, the measurement of $g^{(2)}(\tau)$ can be considered as an alternative to the technique based on using a multichannel analyzer when spectral lines are being analyzed.

Table IV. - Values of f_α = error in α from $g^{(2)}(\mathbf{T})$ / error in α from multichannel analyzer.

$\Delta t/P \approx 0.2$	$h \approx 0.5$	$\Delta t/P \approx 0.2$	$t_1/P \approx 0.3$	$h \approx 0.5$	$t_1/P \approx 0.3$
t_1/P	f_A	f_h	f_{t_1}	h	f_A
0.1	1.73	0.85	1.09	0.1	0.87
0.3	0.68	1.24	0.88	0.3	1.12
0.5	0.67	1.61	3.70	0.5	0.69
0.7	0.54	1.07	3.70	0.7	0.67
0.9	1.01	1.26	2.56	0.9	0.60
$\langle f_\alpha \rangle$	0.93	1.20	2.4	$\langle f_\alpha \rangle$	0.79

t_1/P	f_A	f_h	f_{t_1}	h	f_A	f_h	f_{t_1}	$\Delta t/P$	f_A	f_h	f_{t_1}
0.1	1.73	0.85	1.09	0.1	0.87	1.16	0.86	0.10	0.47	0.83	0.72
0.3	0.68	1.24	0.88	0.3	1.12	0.75	1.37	0.15	0.69	0.83	0.73
0.5	0.67	1.61	3.70	0.5	0.69	1.24	0.88	0.20	0.69	1.20	0.88
0.7	0.54	1.07	3.70	0.7	0.67	1.26	1.01	0.25	0.85	1.10	0.83
0.9	1.01	1.26	2.56	0.9	0.60	3.47	0.72	0.30	1.07	3.60	1.4
$\langle f_\alpha \rangle$	0.93	1.20	2.4	$\langle f_\alpha \rangle$	0.79	1.60	0.97	$\langle f_\alpha \rangle$	0.75	1.5	0.91

The results reported in this work are basically the same as those obtained for an intensity composed of two spectral lines with Lorentzian profile. Therefore, the behavior of the errors involved in analyzing two spectral lines with Voigt profile is expected to be similar to those obtained in above-mentioned profiles studies.

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